

# Mean field baryon magnetic moments and sumrules

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PACS. 11.15.Pg – Expansions for large numbers of components (e.g.,  $1/N_c$  expansions) .

PACS. 11.55.Hx – Sum rules.

PACS. 12.39.-x – Phenomenological quark models.

## Abstract. –

New developments have spurred interest in magnetic moments ( $\mu$ -s) of baryons. The measurement of some of the decuplet  $\mu$ -s and the findings of new sumrules from various methods are partly responsible for this renewed interest. Our model, inspired by large colour approximation, is a relativistic self consistent mean field description with a modified Richardson potential and is used to describe the  $\mu$ -s and masses of all baryons with up (u), down (d) and strange (s) quarks. We have also checked the validity of the Franklin sumrule (referred to as CGSR in the literature) and sumrules of Luty, March-Russell and White. We found that our result for sumrules matches better with experiment than the non-relativistic quark model prediction. We have also seen that quark magnetic moments depend on the baryon in which they belong while the naive quark model expects them to be constant.

*Introduction.* – 't Hooft suggested that the inverse of number of colours ( $N_c$ ) could be used as an expansion coefficient in the otherwise parameter free QCD [1]. Based on this, Witten [2] suggested mean field (MF) description for baryons - prompting the use of phenomenological interquark potential tested in meson sector. Indeed baryon mass ( $M$ ) was calculated at MF level [3] using Richardson potential as an interquark one [4]. The potential has confinement and asymptotic freedom (AF) built in - with a single scale parameter ( $\Lambda \sim 400 \text{ MeV}$ ). Although the lattice QCD gives a confinement scale ( $\sim 400 \text{ MeV}$ ) - the AF scale as given by perturbative QCD is  $\sim 100 \text{ MeV}$ . For computing the hadron properties a  $\Lambda \sim 400 \text{ MeV}$  is required whereas for high density strange quark matter (SQM in short)  $\Lambda \sim 100 \text{ MeV}$  [5]. This is not surprising as for the SQM the confinement gets screened and a much reduced  $\Lambda$  appropriate for AF part is required. Hence it is important to separate out the two scales and re-do the hadron properties -like magnetic moments and masses for the baryons. In fact, Bagchi et al [6] calculated the masses ( $M$ ) and magnetic moments  $\mu$  of  $\Delta^{++}$  and  $\Omega^-$  with a modified two parameter Richardson potential using relativistic Hartree-Fock (RHF) method.

The known  $\mu$ -s cannot be explained by any model exactly and in addition there are some sumrules which are interesting to look at. The first one that is much referred is the Franklin sumrule, named erroneously after Coleman and Glashow [7]. The latter authors derived mass

sumrules for SU(3) breaking and found specific  $\mu$ -s but not the actual sumrule for the octet  $\mu$ -s - which was first done by Franklin [8] and so should carry his name<sup>(1)</sup>.

Presently, extending the calculation of Bagchi et al [6] with the modified potential we find all baryonic (decuplet and octet)  $\mu$ -s and check few sumrules to test symmetry assumptions. We compare with experiment and other calculations hoping more experimental decuplet  $\mu$ -s to be soon deduced. Our values are thus predictions. One can view our calculation in the spirit of large  $N_c$  or consider it as a relativistic MF calculation with a potential having AF and confinement property built into it. We compare our results with those obtained from analytic large  $N_c$  formulations which are exact calculations but assumes static limit as  $M$  is infinite for  $N_c = \infty$ . Our  $M$ -s are finite, so the comparison is interesting - displaying the effect of the wave functions and whatever dynamics it contains.

*Formalism.* - One needs to sum all planar gluon exchange diagrams to deduce an effective interquark potential. Analytic derivation of such a sum being absent, a potential like Richardson potential is chosen from meson phenomenology and then tested for baryons and quark stars [3, 5]

$$V(r) = -\frac{N_c + 1}{2N_c} \frac{6\pi}{33 - 2N_f} [\Lambda^2 r - f(\Lambda r)] \quad (1)$$

$-\frac{N_c+1}{2N_c}$  is colour contribution,  $N_c$  is number of colours ( $= 3$ ),  $N_f$  is number of flavours ( $= 3$ ).

$$f(t) = 1 - 4 \int_1^\infty \frac{dq}{q} \frac{\exp(-qt)}{[\ln(q^2 - 1)]^2 + \pi^2} \quad (2)$$

Richardson calculated, non-relativistically, the masses of two heavy mesons,  $J/\Psi$  and  $\Upsilon$  [4]. Crater and Van Alstine [9] obtained masses of both light and heavy mesons using a relativistic two body Dirac equation with  $\Lambda = 401 \text{ MeV}$ . Dey *et. al.* [3] calculated the baryonic properties like  $M$  and  $\mu$  of  $\Omega^-$  with  $\Lambda = 400 \text{ MeV}$  using RHF method. However, when the potential was used in strange star calculation [5] the required  $\Lambda \sim 100 \text{ MeV}$ . Strange stars are very compact objects composed of high density SQM. Debye screening length of the gluon suppresses the confinement due to the medium effect and a lower  $\Lambda$  is sufficient.

Separating the confinement and AF scales the modified potential is given by

$$V(r) = -\frac{N_c + 1}{2N_c} \frac{6\pi}{33 - 2N_f} [\Lambda'^2 r - f(\Lambda r)] \quad (3)$$

$\Lambda' = 350 \text{ MeV}$  (confinement) and  $\Lambda = 100 \text{ MeV}$  (AF) gave satisfactory results for the hadrons  $\Delta^{++}$  and  $\Omega^-$  [6]. Also the strange star calculation improves [10], - increasing the value of the strong coupling constant  $\alpha_s$  in Debye screening term. This is more consistent in the framework of the star calculation in view of the findings of [11] where the density dependence of quark masses is found from  $\alpha_s$ .

We set out to apply this potential to find the  $\mu$ -s of all other baryons, and check magnetic moment sumrules, with no more extra parameter to adjust.

*Details of Calculation.* - The Quark wave function for quarks in lowest ( $1s_{1/2}$ ) orbital

$$\phi_q(r) = \left[ \frac{1}{4\pi} \right]^{\frac{1}{2}} \begin{pmatrix} iG(\vec{r})\chi_m \\ \vec{\sigma} \cdot \hat{r} F(\vec{r})\chi_m \end{pmatrix}; \quad q = u, d, s \quad (4)$$

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<sup>(1)</sup>We are grateful to Jerry Franklin for pointing this out to us and allowing us to correct the mistake.

$\chi_m$  is Pauli spinor,  $\vec{\sigma}$  is Pauli matrix. The Hamiltonian and corresponding HF equations are:

$$H = \sum_i^3 t_i + \sum_{i < j} V(r_{ij}) \quad (5)$$

$$i = 1, 2, 3; \quad j = 1, 2, 3; \quad t_i = \vec{\alpha} \cdot \vec{p}_i + \beta m_i \quad (6)$$

$$[t_q + \omega_q(r_1)] \phi_q(r_1) = \epsilon_q \phi_q(r_1) \quad (7)$$

$t_i$  is the kinetic energy operator for the  $i$ th quark,  $V(r_{ij})$  is the modified potential,  $\epsilon_q$ -s are the single particle energies and  $\omega_q$ -s are the single particle potentials.

$$\omega_{q1}(r_1) = \int \phi_{q2}^\dagger(r_2) V(r_{12}) \phi_{q2}(r_2) r_2^2 dr_2 + \int \phi_{q3}^\dagger(r_3) V(r_{13}) \phi_{q3}(r_3) r_3^2 dr_3 \quad (8)$$

Using the wave functions given in eqn. [4], we get sets of coupled differential equations

$$\begin{aligned} \frac{dG_q}{dr} - (m_q - \omega_q + \epsilon_q) F_q &= 0 \\ \frac{dF_q}{dr} + \left(\frac{2}{r}\right) F_q + (\epsilon_q - \omega_q - m_q) G_q &= 0 \end{aligned} \quad (9)$$

From  $\epsilon_q$ -s, the energy is obtained from the following equation

$$\begin{aligned} E = \epsilon_u + \epsilon_d + \epsilon_s - \frac{1}{2} \int \phi_u^\dagger(r_1) \omega_u \phi_u(r_1) r_1^2 dr_1 - \\ \frac{1}{2} \int \phi_d^\dagger(r_2) \omega_d \phi_d(r_2) r_2^2 dr_2 - \frac{1}{2} \int \phi_s^\dagger(r_3) \omega_s \phi_s(r_3) r_3^2 dr_3 \end{aligned} \quad (10)$$

Here the subtractions are to avoid double counting which comes when applying variational principle to derive HF equations.

The coupled differential equations are to be solved self consistently. Procedure is to check the convergence in the energy value. Moreover, to make the calculation more transparent and easier we take recourse to expansion of wave functions in oscillators and subsequent diagonalisation,

$$G(r) = \sum_n C_n R_{n0} \quad (11)$$

$$F(r) = \sum_m D_m R_{m1} \quad (12)$$

$C$ -s and  $D$ -s are coefficients. This reduces the differential equations to an eigenvalue problem. Starting with a trial set of  $C$ -s and  $D$ -s, the solution is found self consistently by diagonalizing the matrix and putting back the coefficients till convergence is reached. In general  $R_{nl}(r)$  is

$$R_{nl}(r) = \sqrt{\frac{2n!}{\Gamma(n+l+\frac{3}{2})}} r^l \exp(-\frac{1}{2}r^2) L_n^{l+\frac{1}{2}}(r^2) \quad (13)$$

$L_n^{l+\frac{1}{2}}(r^2)$  are Associated Laguerre polynomials. In the calculation,  $r$  is replaced by  $r/b$ ,  $b$  being the oscillator length which may be different for  $G(r)$  and  $F(r)$  -  $b$  and  $b'$ , respectively.

The centre-of-mass (CM) momentum is not well-defined in RHF solutions and this entails a spurious contribution from the CM kinetic energy to the total energy. Since the relative importance of this effect increases as the number of particles decreases, it is necessary to correct it for systems formed of few particles. This has been done here by extending the Peierls-Yoccoz procedure of nuclear physics. The spurious contribution is denoted by  $T_{CM}$  and the baryon mass  $M$  is  $E - T_{CM}$ . The Peierls-Yoccoz procedure corrects the energy but for  $\mu$  - correct boosted wave functions are needed. This is discussed in the reference [12]. Boosting is not attempted in the present paper since our ultimate aim is to use the procedure for a large system, a massive star, where these corrections are not relevant. Therefore, our results on masses of decuplet and octet baryons, take care of center of mass correction but not for the magnetic moments. Still, we are able to compare our calculated  $\mu - s$  with infinite mass static large  $N_c$  models which also do not involve centre of mass correction<sup>(2)</sup>.

Energy differences between decuplet and octet baryons are obtained using the simplest idea of instanton physics [14] where  $\alpha$  is the instanton induced potential between a (u,d) pair and  $\beta$  is the same between a (u, s) or (d, s) pair.

$$E_N = E_\Delta - 3\alpha/2 \quad (14)$$

$$E_\Lambda = E_\Lambda - \alpha - \beta/2 \quad (15)$$

$$E_\Sigma = E_{\Sigma^*} - 3\beta/2 \quad (16)$$

$$E_\Xi = E_{\Xi^*} - 3\beta/2 \quad (17)$$

There are alternative methods also *e.g.* incorporating colour magnetic interaction energy [15].

The r.m.s. radii  $r_{av}$  can also be estimated using the expression

$$r_{av} = \sqrt{\frac{1}{3} \int_0^{r_{max}} [(G_u(r)^2 + F_u(r)^2) + (G_d(r)^2 + F_d(r)^2) + (G_s(r)^2 + F_s(r)^2)] r^4 dr} \quad (18)$$

But as the potential is spin independent, the radii for octet and decuplet members become the same.

In quark model, the magnetic moment associated with a quark is given by [16]:

$$\mu_q = \frac{e_q}{2} \int (\vec{r} \times \vec{j}) d^3r \quad (19)$$

$e_q$  is the charge of the quark,  $j$  is the current associated with the quark.  $\mu_q$  reduces to :

$$\mu_q[\uparrow (\downarrow)] = - (+) e_q \frac{2}{3} \int_0^\infty G(r) F(r) r^3 dr \quad (20)$$

Baryonic  $\mu$  is found using baryonic wave functions  $\Psi_{spin} \times \Psi_{flavor}$  [17].

*Results.* - Subtracting  $T_{CM}$  from HF energy  $E$ , we obtained the baryonic masses  $M$  which are given in table I along with the experimental values.  $T_{CM}$  lies  $\sim 100$  MeV for all the baryons. We have chosen  $\alpha = 188$  and  $\beta = 103$  MeV [14] which gives the overall best fit. The values of the oscillator parameters  $b$  and  $b'$  are chosen such that  $E$  becomes independent of variation of  $b$ ,  $b'$ . There are no other free parameters to fit. We have also found that  $r_{av}$  is  $\sim fm$  and it decreases with increasing  $M$ .

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<sup>(2)</sup>Since an oscillator basis is used for the calculation, it is possible to do standard CM correction exemplified by Elliott and Skyrme [13].

TABLE I – Charge averaged baryon masses (both decuplet and octet members) where  $\Lambda'$  is 350 MeV and  $\Lambda$  is 100 MeV; using  $7 \times 7$  matrices. The quark masses are  $m_u, m_d \sim 10$  MeV,  $m_s = 150$  MeV. We have chosen  $\alpha = 188$  MeV and  $\beta = 103$  MeV for a good fit of nucleon and  $\Sigma$  comparing to the experimental value.

| Baryons       | Experimental Mass<br>(MeV) | Theoretical Mass<br>(MeV) | $b$  | $b'$ |
|---------------|----------------------------|---------------------------|------|------|
| $\Delta$ 's   | 1232                       | 1251                      | 0.83 | 0.60 |
|               | 1383                       |                           |      |      |
| $\Sigma^*$ 's | 1384                       | 1361                      | 0.83 | 0.60 |
|               | 1387                       |                           |      |      |
| $\Xi^*$ 's    | 1532                       | 1455                      | 0.77 | 0.60 |
|               | 1535                       |                           |      |      |
| $\Omega^-$    | 1672                       | 1556                      | 0.70 | 0.60 |
|               |                            | 938                       |      |      |
| $N$           | 939                        | 938                       | 0.83 | 0.60 |
|               | 1189                       |                           |      |      |
| $\Sigma$ 's   | 1193                       | 1188                      | 0.83 | 0.60 |
|               | 1197                       |                           |      |      |
| $\Lambda^0$   | 1116                       | 1098                      | 0.83 | 0.60 |
|               | 1315                       |                           |      |      |
| $\Xi$ 's      | 1321                       | 1282                      | 0.77 | 0.60 |

Table II shows a comparison between our  $\mu$ -s with experimental and other theoretical values. The agreement of  $M$ -s and  $\mu$ -s from our result with those from experiments is not too unreasonable. Results obtained in QCD sum rule (QCDSR) approach are taken from [18] for the decuplet and from [19] for the octet. Results obtained by another large  $N_c$  approximation are taken from [20] where the authors fit the octet and the  $\Omega^-$   $\mu$ -s to predict the other decuplet  $\mu$ -s. Experimental values are taken from [21]. We have also compared our result with lattice [22] and chiral perturbation theory,  $\chi pt$  [23]. Dai *et. al* [24] fitted the octet  $\mu$ -s, in their fit A and fit B, by adjusting 10 parameters and then predicted the unknown  $\mu$ -s. For the decuplet the agreement between the calculations and the three known  $\mu$ -s are the only guides.

The new experimental value of  $\mu_{\Delta^{++}}$  agrees better with our result, QCDSR and lattice than the others. The Franklin sumrule [8] mentioned in the introduction is as follows :

$$(\mu_p - \mu_n) + (\mu_{\Sigma^-} - \mu_{\Sigma^+}) + (\mu_{\Xi^0} - \mu_{\Xi^-}) = \Delta_{Franklin} = 0. \quad (21)$$

The value of  $\Delta_{Franklin}$  is +0.48 from experimental  $\mu$ -s, +0.47 from CDM [25]  $\mu$ -s, +0.14 and +0.30 from Franklin's recent calculation [26] and +0.45 from our results.

Taking infinite colour limit and the consequent static infinite mass one can get some other sumrules among the  $\mu$ -s [27, 28]. Luty *et al.* [28] coupled the mass expansion for the s quark to this limit which makes it interesting to compare with our calculation (table III). Our results seem to be intermediate to the sumrules and experiment.

With help of equation (20) we found that  $\mu$  for a particular quark changes in different baryons, whereas quark  $\mu$ -s are constant in naive quark model;  $\mu_u = +1.852$ ,  $\mu_u = -0.972$  and  $\mu_s = -0.613$  [21]. We found that with increasing M, the magnitude of quark  $\mu$  decreases (see table IV).

TABLE II – Comparison of baryon magnetic moments found in different approaches.

|                | $\Delta^{++}$ | $\Delta^+$ | $\Delta^0$ | $\Delta^-$ | $\Sigma^{*+}$ | $\Sigma^{*0}$         | $\Sigma^{*-}$ | $\Xi^{*0}$ | $\Xi^{*-}$ | $\Omega^-$ |
|----------------|---------------|------------|------------|------------|---------------|-----------------------|---------------|------------|------------|------------|
| Ours           | 5.77          | 2.88       | 0.         | -2.86      | 2.81          | 0.17                  | -2.46         | +0.30      | -2.17      | -1.92      |
| Expt. [21]     | 6.14          | 2.70       | -          | -          | -             | -                     | -             | -          | -          | -2.02      |
| QCDSR [18]     | 6.14          | 3.02       | 0.0        | -3.07      | 1.90          | -0.07                 | -2.03         | 0.80       | -2.71      | -2.02      |
| lattice [22]   | 6.09          | 3.05       | 0.0        | -3.05      | 3.16          | 0.33                  | -2.5          | 0.58       | -2.08      | -1.73      |
| $\chi pt$ [23] | 4.0           | 2.1        | -0.17      | -2.25      | 2.0           | -0.07                 | -2.2          | 0.10       | -2.0       | -          |
| $1/N_c$ [20]   | -             | 3.04       | 0.0        | -3.04      | 3.35          | +0.32                 | -2.79         | 0.64       | -2.36      | -          |
| Dai fit A [24] | 5.84          | -          | -          | -          | -             | -                     | -             | -          | -          | -2.08      |
| Dai fit B [24] | 5.86          | -          | -          | -          | -             | -                     | -             | -          | -          | -2.06      |
|                |               | p          | n          |            | $\Sigma^+$    | $\Sigma^0, \Lambda^0$ | $\Sigma^-$    | $\Xi^0$    | $\Xi^-$    |            |
| Ours           |               | 2.88       | -1.91      |            | 2.59          | 0.83, -0.71           | -0.92         | -1.45      | -0.62      |            |
| Expt. [21]     |               | 2.79       | -1.91      |            | 2.46          | -0.61                 | -1.16         | -1.25      | -0.65      |            |
| QCDSR [19]     |               | 3.04       | -1.79      |            | 2.73          | -0.50                 | -1.26         | -1.32      | -0.93      |            |
| $CDM$ [25]     |               | 2.79       | -2.07      |            | 2.47          | -0.71                 | -1.01         | -1.52      | -0.61      |            |
| Dai fit A [24] |               | 2.84       | -1.87      |            | 2.46          |                       | -1.06         | -1.28      | -0.61      |            |
| Dai fit B [24] |               | 2.80       | -1.92      |            | 2.46          |                       | -1.23         | -1.26      | -0.63      |            |

*Conclusions.* – The property of all baryons has been investigated with an improved Richardson potential in a tree level calculation in the large  $N_c$  spirit and the results agree reasonably with experiments and other theoretical ones. The calculation is simple but it gives some of the quark dynamics which is absent in the exact calculation in infinite  $N_c$  (infinite mass) static baryon model. Accurate determination of other decuplet  $\mu$ -s will be helpful for testing models including ours which can be used to predict the equation of state for SQM.

A different approximate relativistic many body method is perhaps possible for three quark systems but would not be relevant for us. We aim to test the validity of MF approach suggested by Witten [2] “QCD simplifies as  $N$  becomes large, and there exists a systematic expansion in powers of  $1/N$ . In various ways, to be discussed later, this expansion is reminiscent of known phenomenology of hadron physics, indicating that an expansion in powers of  $1/N$  may be a good approximation at  $1/N = 3$ .” and “The large  $N$  limit is, instead, given by a sort of Hartree approximation. The logic behind this approximation is as follows. For large  $N$  the interaction between any given pair of quarks is negligible - of order  $1/N$ . But the total potential experienced by any one quark is of order one, since any quark interacts with  $N$  other quarks, each with strength  $1/N$ . Thus, the total potential experienced by any one quark is

TABLE III – Checking of six large  $N_c$  analytic sumrules given by eqn (26) and (30) in Luty, March-Russell and White [28] with our results.

| Large $N_c$ analytic relations   | Our results | Experimental |
|--|-------------|--------------|
| (i) $\mu_p + \mu_n + \mu_{\Sigma^-} = 0$   | +0.05       | -0.28        |
| (ii) $\mu_{\Xi^0} - 2\mu_{\Xi^-} = 0$  | -0.21       | +0.05        |
| (iii) $\mu_{\Xi^-} + \mu_{\Sigma^+} - \mu_{\Sigma^-} - \mu_p = 0$  | +0.01       | +0.18        |
| (iv) $\mu_{\Omega^-} - \mu_{\Xi^0} - \mu_{\Xi^-} = 0$  | +0.15       | -0.12        |
| (v) $\mu_{\Xi^0} + 2\mu_{\Sigma^+} + 2\mu_{\Sigma^-} + \mu_n = 0$  | -0.02       | -0.56        |
| (vi) $\mu_{\Omega^-} + 4\mu_{\Xi^0} - 3\mu_{\Xi^-} + 8\mu_{\Sigma^+} + 5\mu_{\Sigma^-} - 3\mu_p + \mu_n = 0$ | -0.29       | -1.486       |

TABLE IV – *Quark magnetic moments in our calculation*

|         | $\Delta^{++}$ | $\Delta^+$ | $\Delta^0$ | $\Delta^-$ | $\Sigma^{*+}$ | $\Sigma^{*0}$ | $\Sigma^{*-}$ | $\Xi^{*0}$ | $\Xi^{*-}$ | $\Omega^-$ |
|---------|---------------|------------|------------|------------|---------------|---------------|---------------|------------|------------|------------|
| $\mu_u$ | +1.92         | +1.92      | +1.91      | -          | +1.76         | -             | -             | 1.64       | -          | -          |
| $\mu_d$ | -             | -0.96      | -0.95      | -0.95      | -             | -             | -0.87         | -          | -0.82      | -          |
| $\mu_s$ | -             | -          | -          | -          | -0.72         | -             | -0.72         | -0.67      | -0.67      | -0.64      |

of order one, but is a sum of many small, separately insignificant terms. As in statistical mechanics, when a quantity is a sum of many insignificant terms, the fluctuation around the mean value are very small. Thus, the potential experienced by one quark, apart from being of order one, can be regarded as a background, c-number potential-the fluctuations are negligible. To find the ground state baryon, each quark should be placed in the ground state of the average potential that it experiences. By symmetry, the average potential is the same for each quark, so we should place each quark in the same ground state of the average potential". The beauty of MF approximation is that it can be used in both 3 quark system (baryon) and many quark system (quark stars). This has inspired our work.

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